Geometric Modeling of Vehicle Paths and Confidence Regions

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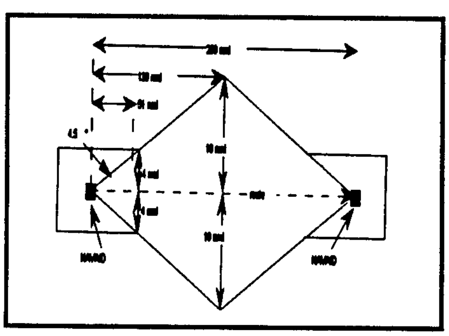
Technical Report No. 127
May, 1996

Center for Computational Statistics



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Geometric Modeling of Vehicle Paths and Confidence Regions

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Abstract

In transportation systems today, there is a need to predict where a vehicle will be at a given time in order to ensure safety, expediency and efficiency of traffic movement. There is generally a plan of travel, but outside forces (e.g., wind forecasting error, navigation system error) cause the actual path that is followed to be somewhat different from the planned path. The path of a vehicle is represented as a vector-valued curve in three-space. The construction of the confidence region about the curve takes advantage of an assumption that the deviation of the actual path from the predicted path will satisfy the conditions for a conditioned Brownian motion process. Using a cubic spline to estimate the predicted path, it is possible to obtain parameter values for the conditioned Brownian motion process, as well as error bounds for constructing the confidence region. An example is given to illustrate the forecasting technique, showing good results in predicting the path, constructing the confidence region, and detecting when the actual distribution of the deviation differs from the estimated distribution.

1. Introduction

There is a need to predict where a vehicle will be at a given time in order to ensure safety, expediency and efficiency of traffic movement. The location of the vehicle can be described in four dimensions: lateral (specified in latitude and longitude), vertical (a distance above ground level or sea level), and longitudinal (time). The lateral dimension is here expressed as a composite of two dimensions. In rail systems, the lateral and vertical dimensions are easily predicted, leaving only time uncertainty. In automotive systems (e.g., busses), vertical dimensions are uninteresting, lateral dimensions can be well-defined (bus routes) or not (personal automobiles), and time is uncertain. In certain amphibian systems (e.g., ferries), there is more uncertainty in the lateral and time dimensions, but still the vertical dimension is uninteresting. However, in underwater or air travel, there can be large uncertainty in all four dimensions. In these cases, there is generally a plan of travel which is followed, but outside forces (wind forecasting error, navigation system error) cause the actual path that is followed to be somewhat different from the planned path.

Path prediction has tended to emphasize short-term, or tactical, path prediction of one vehicle at a time, usually in the arena of military intelligence (e.g., the prediction of the near-term positions of a submarine belonging to the opposing force). The position of an aircraft in the air traffic control system is predicted for two to three minutes in the future and checked against the short-term predicted future positions of other aircraft in the vicinity in order to determine if separation minima are likely to be violated. The short-term predicted position of the aircraft is based upon its reported current position and on a small number of immediately-previous reports, plus knowledge of whether the aircraft is near a planned route bend and the direction of turn (left or right). The long-term prediction is made only for a limited number of selected positions on the planned route and is made based on the planned route and the planned cruising speed. In tactical path prediction, past reports can be used to improve the prediction, by the use of techniques such as a Kalman filter. The extended Kalman filter and the adaptive Kalman filter have also been used as a means of updating the predicted path of a vehicle, especially in the design of control systems. (See Lefas and Thomas, 1981, and Brown, 1983.) These filters are especially appropriate when a vehicle has an extended mission over several months and when initial uncertainties and sparse measurements are not critical factors (Brown, 1983, pp. 300-307).

In this paper, the path of a vehicle is represented as a vector-valued function. A confidence region is constructed about the planned path, based upon previous

traversals of the same path by the same vehicle or other similar vehicles, or upon assumptions about error limits. The construction of the confidence region takes advantage of an assumption that the deviation of the actual path from the planned path will satisfy the conditions for a conditioned Brownian motion process. Using a cubic spline to estimate the predicted path, it is possible to obtain parameter values for the conditioned Brownian motion process, as well as error bounds for constructing the confidence region. An example is given to illustrate the forecasting technique, showing good results in predicting the path, constructing the confidence region, and detecting when the actual distribution of the deviation differs from the estimated distribution.

2. Path Modeling

We introduce the following terminology for the movement of the object. The *planned path* of the object represents the idealized planned movement of the object in the future, defined in terms of points of transition in each spatial dimension. An *actual path* of the object represents the actual positions of the object as measured during one realization of the object's transversal of the planned path. The *mean path* of the object represents the movement of the object "averaged" over many attempts of the object to follow the planned path, i.e., a mean of actual paths. The *predicted path* of the object is the path that the object might be expected to follow during any actual realization of the planned path.

We wish to model the predicted path. In doing so, we view the error in prediction as a Brownian motion process with known points of constriction corresponding to locations which must be traversed (e.g., turns). At the points of constriction, the error is expected to be very small. Thus, we model the error between constriction points as a Brownian bridge. We show that the predicted path can be estimated using a vector spline. It is desired to determine a *confidence region* or envelope about the predicted path. The confidence region represents a volume determined as a function of the error in prediction and within which the object should be located with a given degree of probability.

We represent the planned path as a vector-valued function $\boldsymbol{P}(t) = (p_1(t), \ldots, p_n(t))^T$ of the single real variable t, for $0 < t < T$. We are typically interested in the cases when the dimension n is either two or three, and when the planned path represents an attempt to follow a straight line between adjacent pairs of points (as in the movement of an aircraft). In these cases, when the value of $\boldsymbol{P}$ is known for m distinct values $t_1, \ldots, t_m$, the planned path can be visualized as a connected sequence of straight line segments - the vector-valued function

$$\boldsymbol{P}(t) = \frac{t - t_i}{t_{i+1} - t_i}(\boldsymbol{P}(t_{i+1})) + \frac{t_{i+1} - t}{t_{i+1} - t_i}(\boldsymbol{P}(t_i))$$

for $t \in [t_i, t_{i+1}]$ and $0 \leq t_1 < \ldots < t_m \leq T$ connected at the points $\boldsymbol{P}(t_i)$ (called *joints*). It can be assumed (although this is not essential) that each joint represents a change in direction in one of the spatial dimensions. Because of the abrupt changes in direction at the joints, the planned path belongs at best to C^0.

An actual path $Y(t)$ represents the locations traversed by the object while trying to follow the planned path. The locations observed on the actual path are sampled values.

The mean path, also a vector-valued function, represents the mean of a large number of realizations of the actual path. The mean path $\boldsymbol{m}(t)$ is $\boldsymbol{m}(t) = E[Y_i]$. Each mean path is assumed to be smooth in the sense that any functional representation should have continuous first and second derivatives. That is, we assume any vehicle will move in such a manner that velocity and acceleration will be continuous. We regard an actual path of the object as measurements derived from a model of the predicted path

$$\boldsymbol{Y}(t) = \boldsymbol{m}(t) + \boldsymbol{X}(t)$$

where $\boldsymbol{m}(t)$ is a smooth function and $\boldsymbol{X}(t)$ is the error term. $\boldsymbol{Y}(t)$, $\boldsymbol{m}(t)$ and $\boldsymbol{X}(t)$ are vector valued, i.e.,

$$\boldsymbol{X}(t) = (X_1(t), \ldots, X_n(t))^T$$

$$\boldsymbol{m}(t) = (m_1(t), \ldots, m_n(t))^T$$

$$\boldsymbol{Y}(t) = (Y_1(t), \ldots, Y_n(t))^T$$

Note that the estimation of the curve $\boldsymbol{m}(t)$ is modeled as a vector-valued nonlinear regression problem.

We desire to estimate the mean path by $\hat{\boldsymbol{m}}(t)$. We wish $\hat{\boldsymbol{m}}(t)$ to be "close to" the mean path $\boldsymbol{m}(t)$ and "smooth." By "close to," we mean that a measurement of the difference between $\hat{\boldsymbol{m}}(t)$ and the mean path (the "lack of fit") is small. By "smooth," we mean that each of the $\hat{m}_i(t)$ is "smooth" in the sense of belonging to a Sobolov space. Thus, we require $\hat{\boldsymbol{m}}(t)$ to be a component-wise Sobolov function. Because "closeness" and "smoothness" are competing qualities, we select $\boldsymbol{m}(t)$ to be that component-wise Sobolov function which minimizes a weighted sum of a lack-of-fit term and a smoothing term. This is the general formulation for the classic penalized smoothing spline (see Wegman and Wright, 1983). The difference between our formulation here and that discussed in Wegman and Wright is

that we have a vector-valued spline and in the present problem, we have in mind a more complex error structure than simple i.i.d. random variables.

We assume that the error term behaves in accordance with Brownian motion: that is, 1) the error changes continuously over time and the error at any particular instant of time appears to be normally distributed, 2) the error over any interval depends on the length of the interval rather than on the placement of the interval itself; and 3) the error in any two disjoint intervals is independent. We also assume: 4) that the expected error at any time t is zero. Finally, we assume: 5) that the error term has greater variability between each adjacent pair of joints than at the joints themselves. The latter assumption is crucial. For example, an aircraft typically flies between waypoints on its route of flight. A goal of the pilot, perhaps aided by cockpit automation, is to cross the waypoints. However, between two consecutive waypoints, the aircraft can vary somewhat from a straight line, due to errors in ground- or space-based navigation aids, on-board navigation equipment, and pilotage. Thus, while the object may "wander" somewhat from the planned path on the interior of each line segment, the object is expected to return "close to" the planned path at each joint.

More specifically, we assume that the error term behaves as if it were a Brownian bridge between the joints t_i and t_{i+1}, that is, that $\boldsymbol{X}(t)$ is a Brownian motion process conditioned so that $\boldsymbol{X}(t_i) = \boldsymbol{X}(t_{i+1}) = 0$. The covariance structure of $\boldsymbol{X}$ between the joints t_i and t_{i+1} is of the form

$$cov[\boldsymbol{X}] = \boldsymbol{B} \otimes \Sigma$$

where $\otimes$ is the left Kronecker product and $\boldsymbol{B}$ is a positive definite matrix. We wish to find the covariance structure of $\boldsymbol{X}$ between its first and last joints t_1 and t_m. Because the error term is a Brownian motion process, its behavior between two adjacent joints is independent of its behavior between any two other adjacent joints (i.e., the process has orthogonal increments). Thus the covariance structure

$$\hat{\Sigma} = (\hat{\Sigma}_{i,j})$$

of $\boldsymbol{X}$ between the first and last joints t_1 and t_m is the block diagonal matrix

$$\hat{\Sigma} = \begin{pmatrix} cov[X_{1,1}] & 0 & \cdots & \cdots & 0 \\ 0 & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & cov[X_{i,i}] & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & \cdots & 0 & cov[X_{m,m}] \end{pmatrix}$$

where each $cov[X_{i,i}]$ is defined by

$$\begin{pmatrix} cov[X(t_i^1), X(t_i^1)] & \cdots & cov[X(t_i^1), X(t_i^{n_i})] \\ \vdots & \ddots & \vdots \\ cov[X(t_i^{n_i}), X(t_i^1)] & \cdots & cov[X(t_i^{n_i}), X(t_i^{n_i})] \end{pmatrix}$$

for $t_i \le t_i^1 < \ldots < t_i^{n_i} \le t_{i+1}$. Now $cov[X_{i,j}] = 0$ when $i \neq j$ because of the independent behavior. From the standard theory of conditioned Brownian motion, we know that each $cov[X_{i,i}]$ is of the form

$$cov[x_{i,i}] = (\sigma_i^2 \frac{(t_i^k - t_i)(t_{i+1} - t_i^l)}{t_{i+1} - t_i})$$

for $t_i \le t_i^k \le t_i^l \le t_{i+1}$. Letting

$$b_{i_k,i_l} = \frac{(t_i^k - t_i)(t_{i+1} - t_i^l)}{t_{i+1} - t_i},$$

$$\boldsymbol{B}_{i,i} = (b_{i_k,i_l}),$$

$$\boldsymbol{B} = diag(\boldsymbol{B}_{i,i})$$

$$(\hat{\hat{\Sigma}}_{i,i}) = \sigma_i^2 \boldsymbol{I}$$

and

$$\hat{\hat{\Sigma}} = diag(\hat{\hat{\Sigma}}_{i,i})$$

with $\boldsymbol{I}$ being the $i_n \times i_n$ identity matrix, we can write

$$cov[\boldsymbol{X}] = \boldsymbol{B} \otimes \hat{\hat{\Sigma}}$$

as the covariance matrix $cov[\boldsymbol{X}]$ between the first and last joints t_1 and t_m where $\boldsymbol{B}$ is a positive definite matrix. Taking advantage of the covariance structure between the joints, a natural lack-of-fit term is

$$\sum_{i=1}^{n} \sum_{j=1}^{n} b^{i,j} (Y_i - m(t_i))^T \Sigma^{-1} (Y_j - m(t_j)),$$

where $\boldsymbol{Y} = (\boldsymbol{Y}_1, \boldsymbol{Y}_2, \ldots, \boldsymbol{Y}_n)^T$, $\boldsymbol{B}^{-1} = (b^{i,j})$, and T denotes transpose.

Since $\boldsymbol{m}(t)$ is vector-valued,

$$\int (L(\boldsymbol{m}(t)))^T (L(\boldsymbol{m}(t))) dt = \int \sum_{i=1}^{n} (L(m_i(t)))^2 dt$$

is appropriate as a smoothing term.

Thus, we formulate the following minimization problem: Choose $\boldsymbol{m}(t)$ to minimize

$$\sum_{i=1}^{n} \sum_{j=1}^{n} b^{i,j} (Y_i - m(t_i))^T \Sigma^{-1} (Y_j - m(t_j))$$

$$+ \quad \nu \int (L(\boldsymbol{m}(t)))^T (L(\boldsymbol{m}(t)))dt$$

where $\boldsymbol{B}$ and Σ are as described and $\nu > 0$. Miller and Wegman (1987) prove that this minimization problem can be solved componentwise, producing a vector spline which is a vector of scalar splines. They show that the objective function may be rewritten as

$$\sum_{i=1}^{p} \frac{1}{\lambda_i} (\sum_{j=1}^{n} \sum_{k=1}^{n} b^{jk} (W_{ji} - Z_i(t_j))(W_{ki} - Z_i(t_k))$$

$$+ \quad \nu \int (L(Z_i(t)))^2 dt)$$

where $\Sigma = \boldsymbol{P} \Lambda \boldsymbol{P}^T$ is the spectral decomposition (i.e., $\boldsymbol{P}$ is orthogonal and $\Lambda = diag(\lambda_i)$ is a diagonal matrix), $\boldsymbol{W}_k = \boldsymbol{P}^T \boldsymbol{Y}_k$, and $\boldsymbol{Z}(t) = \boldsymbol{P}^T \boldsymbol{m}(t)$. Since each of the p components is non-negative, the sum is minimized by minimizing each of the components. Note that there are n observations in p-space.

3. Confidence Bands

The confidence region is an envelope around the predicted path. The actual path of the object is expected to be located within the confidence region with a high degree of probability. We are given n observations fitting the model $Y(t) = m(t) + X(t)$. Because a vector-valued minimization problem can be solved component-wise, we can assume here that $Y(t)$, $m(t)$, and $X(t)$ are scalar valued. We assume that the error term $X(t)$ is a conditioned Brownian motion process (a Brownian bridge) between each pair of points between $[a, b]$; that is, for each t between a and b, $X(t)$ is normal with mean

$$\mu_t = A + (B - A)\frac{(t-a)}{(b-a)}$$

for some real A and B, variance

$$\sigma_t^2 = \sigma^2 \frac{(b-t)(t-a)}{(b-a)}$$

for some σ^2, and covariance

$$cov[X(s), X(t)] = \sigma^2 \frac{(b-t)(s-a)}{(b-a)}$$

for $a \le s \le t \le b$. In particular, the covariance structure of $X(t)$ is of the form

$$cov[X(t)] = \boldsymbol{B} \otimes \Sigma$$

where $\otimes$ is the left Kronecker product, $\boldsymbol{B}$ is a positive definite matrix, and $\boldsymbol{B}^{-1} = (b^{i,j})$.

The solution to the minimization problem minimize:

$$\sum_{i=1}^{n} \sum_{j=1}^{n} b^{i,j} (y_i - f(x_i))(y_j - f(x_j)) + \lambda \int_a^b (f''(x))^2 dx$$

which takes advantage of the covariance structure of $X(t)$ is a cubic spline $s(x)$. We propose that a $(1-\alpha)$ 100 percent confidence region for f is

$$C(\alpha, x) = s(x) \pm z_{\alpha/2}\sqrt{v_x}$$

where $z_{\alpha/2}$ is the $\alpha/2$ point of the standard normal distribution and v_x is a data-based estimate of the error at x.

4. An Air Traffic Control Example

We apply the theory to the modeling of the path of an aircraft. While the pilot of an aircraft might intend to fly directly from one point to the next, there are many sources of error preventing this direct flight. When the pilot is using an area navigation (RNAV) system designed for use in the U. S. National Airspace System, the sources of navigation system horizontal error are assumed to include ground VOR radiated signal, airborne VOR receiver equipment, area navigation equipment, and pilotage. (See DOT/FAA, 1975.) In addition, it is assumed that the errors attributable to these four sources are independent normal distributions that may be combined root-sum-square (RSS) fashion. (See also ICAO, 1985.) The 2s (95th percentile) values of the error components assumed in the U.S. are summarized in Table 1.

Table 1. Horizontal Error Components of the VOR/DME System

Error Component	2s Error
VOR Equipment	
Ground VOR radiated signal	1.2°
Airborne VOR receiver	2.7°
Course setting equipment	1.6°
Pilotage	2.3°

Suppose that the aircraft is navigating by reference to the ground-based navigation aids (NAVAIDs) of the air traffic control (ATC) system (see FAA, 1992). When the route between two NAVAIDs is not part of an established airway or route and is below flight level 600

(approximately 60,000 feet in altitude), then air traffic controllers must protect the airspace along the route for four miles on each side of the route until 51 miles from the NAVAID and then increasing along a 4.5 degree angle to ten miles on each side at 130 miles from the NAVAID. See Figure 1. Thus, we can begin to estimate the allowable displacement along the route.

The shape of the protected route width suggests that the allowable displacement could be defined as a Brownian motion process conditioned to be (essentially) zero at both ends of the route segment, i.e., at the NAVAIDs. If the displacement at time t is represented by $Y(t)$, then $Y(t)$ is a Brownian motion process conditioned by $Y(t_1) = 0 = Y(t_2)$, when $Y(t_1)$ represents the displacement at time t_1 at the first NAVAID and $Y(t_2)$ represents the displacement at time t_2 at the second NAVAID.

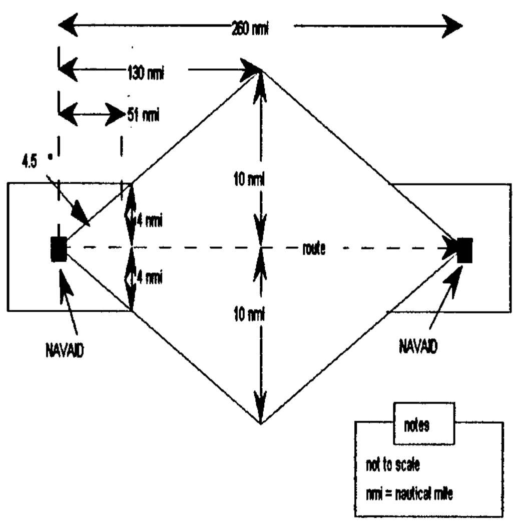

Figure 1. Protected Route Width (ATC)

We know that

$$Y(t) \sim N(0, \sigma^2 \frac{(t_2 - t)(t - t_1)}{t_2 - t_1}).$$

Letting $t_1 = 0$ and $t_2 = 260$, the variance is of the form $\sigma^2(260 - t)t/260$, showing that the largest displacement is possible at the midpoint ($t = 130$) between the two NAVAIDs. We estimate σ by assuming that $1.96\sqrt{\sigma^2(260 - t)t/260}$ represents the 95th percentile of the distribution and solving the equation

$$10 = 1.96\sqrt{\sigma^2(260 - 130) \cdot 130/260}$$

for σ . The solution is $\sigma \sim 0.632830279$. Figure 2 compares two displacement regions on one side of the route: the ATC-defined region and the 95th percentile of the derived distribution.

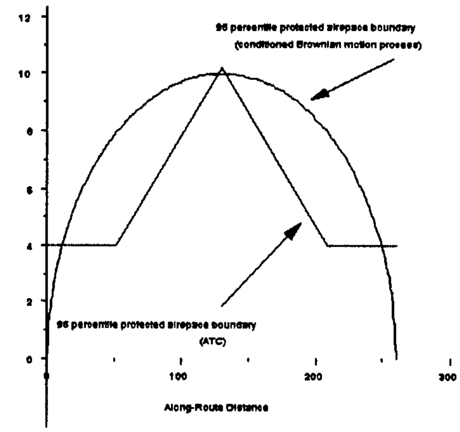

Figure 2. Protected Route Width (Conditioned Brownian Motion Process)

5. Conclusions

In this paper we outline a strategy for the estimation of the path of a vehicle and a confidence region around that path. For an important class of vehicles which includes commercial airliners, the path is a curve in three-dimensional space. This places path modeling in the domain of differential geometry because we wish a path to be a curve in three-space with at least two continuous derivatives (namely velocity and acceleration). A path is modeled as a vector-valued function with two continuous derivatives. We can assume therefore that the path is a curve, an element of a vector-valued Sobolov space. An observed path is modeled as the curve with added noise.

The nature of the additive noise departs from the typical white Gaussian noise assumption both because of a likely correlation in the noise structure and also because the nature of motion towards a destination implies less variability at the origin and destination of the vehicle motion. For this reason we assume a Brownian bridge model for the additive noise. The path being modeled as a vector-valued function with two continuous derivatives taken together with a Brownian bridge error structure implied a fit to the path that is a vector of cubic splines albeit not a traditional cubic spline. Miller and Wegman (1987) develop a theory of vector splines and develop computational algorithms for certain classes of vector splines. These methods apply to the setting in which the error structure is a Brownian bridge.

We conclude the paper with a comparison of the protected route width of the U.S. air traffic control (ATC) system and the estimated 95% confidence band assuming the spline-Brownian bridge model. If the model is valid for aircraft movement, then it can be suggested that current air traffic control procedures reserve too much airspace near navigation aids and too little airspace mid-route. Thus the qualitative shape difference is of concern, and our work suggests that a different model for protected airspace may be appropriate. We note that we predicated our confidence band on matching maximum route widths with the ATC protected route width specification. Because data to actually calibrate a 95% confidence band is not easily available, conclusions about the adequacy of the protected route width in quantitative terms are not warranted. A realistic empirical study of actual aircraft movement along ATC routes would be fruitful since it is possible that an accurately calibrated estimate of an aircraft path and its confidence region could conceivably remove excessively restrictive protected route width specifications. In addition, the confidence region parameter values used in the example are derived from nominal values for navigation accuracy used for all navigation systems as a maximum error budget. However, individual navigation systems are capable of less error. The nominal values, being conservative, require that more airspace be reserved for each aircraft. Using error values tailored to individual navigation systems might also allow better utilization of airspace. Further studies could determine (a library of) error values for the confidence region parameters.

Acknowledgment

The work of Dr. Wegman was supported by the Army Research Office under Grant DAAH04-94-G-0267. The work of Dr. Ball was supported by The MITRE Corporation through its Accelerated Graduate Degree Program in Science and Engineering. This paper is extracted from her dissertation for the Ph.D. degree in Information Technology and Engineering, written under the direction of Dr. Wegman. Dr. Ball is with The MITRE Corporation/Center for Advanced Aviation System Development, 1820 Dolley Madison Blvd, McLean, VA. 22102-3481. The authors thank Shan-Chuan Li for converting this manuscript to TeX form.

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REPORT DOCUMENTATION PAGE

Form Approved
OMB NO. 0704-0188

1. AGENCY USE ONLY *(Leave blank)*	2. REPORT DATE	3. REPORT TYPE AND DATES COVERED
	May 1996	Technical

4. TITLE AND SUBTITLE

Geometric Modeling of Vehicle Paths and Confidence REgions

5. FUNDING NUMBERS

DAAH04-94-G-0267

6. AUTHOR(S)

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7. PERFORMING ORGANIZATION NAMES(S) AND ADDRESS(ES)

Center for Computational Statistics
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8. PERFORMING ORGANIZATION REPORT NUMBER

#127

9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)

U.S. Army Research Office
P.O. Box 12211
Research Triangle Park, NC 27709-2211

10. SPONSORING / MONITORING AGENCY REPORT NUMBER

ARO 32850.15-MA

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13. ABSTRACT *(Maximum 200 words)*

In transportation systems today, there is a need to predict where a vehicle will be at a given time in order to ensure safety, expediency and efficiency of traffic movement. There is generally a plan of travel, but outside forces (e.g., wind forecasting error, navigation system error) cause the actual path that is followed to be somewhat different from the planned path. The path of vehicle is represented as a vector-valued curve in three-space. The construction of the confidence region about the curve takes advantage of an assumption that the deviation of the actual path from the predicted path will satisfy the conditions for a conditioned Brownian motion process. Using a cubic spline to estimate the predicted path, it is possible to obtain parameter values for the conditioned Brownian motion process, as well as error bounds for constructing the confidence region. An example is given to illustrate the forecasting technique, showing good results in predicting the path, constructing the confidence region, and detecting when the actual distribution of the deviation differs from the estimated distribution.

14. SUBJECT TERMS

Navigation errors, conditioned Brownian motion, predicted path, air traffic control

15. NUMBER IF PAGES

8

16. PRICE CODE

17. SECURITY CLASSIFICATION OR REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
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The Report Documentation Page (RDP) is used in announcing and cataloging reports. It is important that this information be consistent with the rest of the report, particularly the cover and title page. instructions for filling in each block of the form follow. It is important to ***stay within the lines*** to meet ***optical scanning requirements.***

Block 1. Agency Use Only *(Leave blank)*

Block 2. Report Date. Full publication date including day, month, and year, if available (e.g. 1 Jan 88). Must cite at least year.

Block 3. Type of Report and Dates Covered. State whether report is interim, final, etc. If applicable, enter inclusive report dates (e.g. 10 Jun 87 - 30 Jun 88).

Block 4. Title and Subtitle. A title is taken from the part of the report that provides the most meaningful and complete information. When a report is prepared in more than one volume, repeat the primary title, add volume number, and include subtitle for the specific volume. On classified documents enter the title classification in parentheses.

Block 5. Funding Numbers. To include contract and grant numbers; may include program element number(s), project number(s), task number(s), and work unit number(s). Use the following labels:

C	- Contract	**PR**	- Project
G	- Grant	**TA**	- Task
PE	- Program Element	**WU**	- Work Unit Accession No.

Block 6. Author(s). Name(s) of person(s) responsible for writing the report, performing the research, or credited with the content of the report. If editor or compiler, this should follow the name(s).

Block 7. Performing Organization Name(s) and Address(es). Self-explanatory.

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- **DOD** - See DoDD 4230.25, "Distribution Statements on Technical Documents."
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